Received: *November 20, 2000,* Accepted: *December 13, 2000*
Revised: *December 13, 2000*

$B \to X_s\gamma$ in supersymmetry: large contributions beyond the leading order

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Abstract: We discuss possible large contributions to $B \to X_s\gamma$, which can occur at the next-to-leading order in supersymmetric models. They can originate from terms enhanced by $\tan\beta$ factors, when the ratio between the two Higgs vacuum expectation values is large, or by logarithm of $M_{\rm SUSY}/M_W$, when the supersymmetric particles are considerably heavier than the W boson. We give compact formulae which include all potentially large higher-order contributions. We find that $\tan\beta$ terms at the next-to-leading order do not only appear from the Hall-Rattazzi-Sarid effect (the modified relation between the bottom mass and Yukawa coupling), but also from an analogous effect in the top-quark Yukawa coupling. Finally, we show how next-to-leading order corrections, in the large $\tan\beta$ region, can significantly reduce the limit on the charged-Higgs mass, even if supersymmetric particles are very heavy.

Keywords: Rare Decays, Supersymmetric Standard Model, NLO Computations.

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JHEP12(2000)009

Contents

1. Introduction

The inclusive radiative decay $B \to X_s\gamma$ provides a powerful experimental testing ground for physics beyond the Standard Model, because of its sensitivity to new particle virtual effects. The measurements of $B \to X_s\gamma$ at CLEO, LEP, and Belle [1], and the progress in the precision expected from experiments at the B factories requires a substantial effort in limiting the uncertainty in the theoretical calculation. This programme has been carried out in the Standard Model up to next-to-leading order corrections [2]–[5], reducing the theoretical error in the prediction for the branching ratio of $B \to X_s\gamma$ to about 10%, equally distributed between renormalization scale dependence and uncertainties in the input parameters [6]. Moreover, non-perturbative effects appear to be under control [7] and several refinements have been introduced in the analysis [6, 8]. In the case of the two-Higgs doublet models, complete next-to-leading order analyses have been presented in ref. [4, 9]. Although the leading-order contributions to $B \to X_s\gamma$ in supersymmetric models are well studied [10], the situation at the next-to-leading order has not been fully analyzed. The next-to-leading order QCD corrections have been calculated [11, 12] assuming minimal flavour violation [13] (i.e. assuming that the Cabibbo-Kobayashi-Maskawa matrix is the only source of flavour violation at the weak scale) and choosing a hierarchical spectrum in which charginos and one stop are lighter than gluinos and the other squarks. In a complementary approach, a systematic leading-order analysis of the contributions from flavour-violating gluino exchanges has been presented in ref. [14], in a very interesting and complete study.

In this paper we want to follow a different approach. We will not try to compute the complete set of next-to-leading corrections in a general supersymmetric model (a rather formidable task), but instead we will identify all potentially large two-loop corrections in models with minimal flavour violation. These come in two classes: (i) corrections enhanced by $\tan\beta$, in the case in which the ratio of the two Higgs vacuum expectation values is large [15]; (ii) corrections enhanced by a logarithm of the ratio $\mu_{\rm SUSY}/\mu_W$, in the case in which the scale of supersymmetric particles $\mu_{\rm SUSY}$ is much larger than the scale μ_W of the W or top mass. In this way, we obtain analytic formulae which, we believe, will give a very good approximation of a complete calculation and, because of their simple form, are very practical to be implemented in analyses of supersymmetric models.

The paper is organized as follows. In section 2 we discuss the terms enhanced by $\tan\beta$ in the next-to-leading corrections to the Standard Model and charged Higgs contributions to the relevant Wilson coefficients. In the limit of very heavy supersymmetric particles, there are two sources of such terms in the charged Higgs contribution. One is coming from the finite corrections to the bottom quark mass (the so-called Hall-Rattazzi-Sarid effect [16]), while the other one is related to its counterpart for the top quark [11]. In section 3 we describe the $\tan^2\beta$ terms appearing at two-loops in the chargino contribution and the log-enhanced contributions in the terms subleading in $\tan\beta$. Section 4 contains a summary of the formulae for the large higher-order contributions to the Wilson coefficients; these formulae can be directly implemented in analyses for $B \to X_s\gamma$. Some numerical results of such an analysis are illustrated in section 5. In particular, we show how next-to-leading order corrections, in the large $\tan\beta$ region, can significantly reduce the limit on the charged-Higgs mass, even if supersymmetric particles are very heavy.

2. Standard Model and charged Higgs contributions

In this paper we are focusing on short-distance contributions and, therefore, we can restrict our discussion to the form of the Wilson coefficients of the $\Delta B = 1$ magnetic and chromo-magnetic operators $Q_7 = (e/16\pi^2) m_b \bar{s}_L \sigma^{\mu\nu} b_R F_{\mu\nu}$ and $Q_8 = (g_s/16\pi^2) m_b \bar{s}_L \sigma^{\mu\nu} t^a b_R G^a_{\mu\nu}$ evaluated at the matching scale μ_W in the effective hamiltonian:

$$\mathcal{H} = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_i C_i(\mu_W) Q_i(\mu_W)\,. \tag{2.1}$$

At the leading order, the contributions to $C_7(\mu_W)$ and $C_8(\mu_W)$ from the Standard Model particles and from the charged Higgs boson are given by

$$\begin{aligned} C_{7,8}^{(SM)}(\mu_W) &= F_{7,8}^{(1)}(x_t) \\ C_{7,8}^{(H^\pm)}(\mu_W) &= \frac{1}{3\,\tan^2\beta} F_{7,8}^{(1)}(y_t) + F_{7,8}^{(2)}(y_t) \end{aligned} \tag{2.2}$$

where

$$x_t = \frac{\bar{m}_t^2(\mu_W)}{M_W^2}\,, \qquad y_t = \frac{\bar{m}_t^2(\mu_W)}{M_H^2}\,. \tag{2.3}$$

Here $\bar{m}_t^2(\mu_W)$ is the SM running top mass and

$$\begin{aligned}
F_7^{(1)}(x) &= \frac{x(7-5x-8x^2)}{24(x-1)^3} + \frac{x^2(3x-2)}{4(x-1)^4}\ln x \\
F_8^{(1)}(x) &= \frac{x(2+5x-x^2)}{8(x-1)^3} - \frac{3x^2}{4(x-1)^4}\ln x \\
F_7^{(2)}(y) &= \frac{y(3-5y)}{12(y-1)^2} + \frac{y(3y-2)}{6(y-1)^3}\ln y \\
F_8^{(2)}(y) &= \frac{y(3-y)}{4(y-1)^2} - \frac{y}{2(y-1)^3}\ln y\,.
\end{aligned} \tag{2.4}$$

The relation between the Wilson coefficients at μ_W and the branching ratio for $B \to X_s\gamma$ is well known (see for example refs. [2, 4]).

The charged Higgs contribution of eq. (2.2) consists of two terms. In the large $\tan\beta$ limit, the first one (in which the chiral flip occurs on the external bottom quark line) is suppressed by $1/\tan^2\beta$, while the second one (in which the chiral flip occurs in the charged Higgs vertex) is independent of $\tan\beta$. The absence of a term enhanced by $\tan\beta$ is a consequence of the fact that, in the large $\tan\beta$ limit, $H^\pm$ decouples from the right-handed top quark. This property is not maintained at the next order in perturbation theory in a supersymmetric model, and thus we expect two-loop charged-Higgs contributions to C_7 and C_8 enhanced by $\tan\beta$.

Let us now extract the $\tan\beta$-enhanced terms. At one-loop, the relation between the bottom quark mass m_b and Yukawa coupling y_b receives a finite correction proportional to $\tan\beta$ [16]:

$$m_b = \sqrt{2}\,M_W\,\frac{y_b}{g}\,\cos\beta\,(1+\epsilon_b\tan\beta)\,. \tag{2.5}$$

The coefficient ϵ_b, generated by gluino-sbottom and chargino-stop diagrams, is given by [16]

$$\epsilon_b = -\frac{2\,\alpha_s}{3\,\pi}\frac{\mu}{m_{\tilde{g}}}H_2(x_{\tilde{b}_1\tilde{g}}, x_{\tilde{b}_2\tilde{g}}) - \frac{y_t^2}{16\,\pi^2}\,\tilde{U}_{a2}\frac{A_t}{m_{\chi_a^+}}\,H_2(x_{\tilde{t}_1\chi_a^+}, x_{\tilde{t}_2\chi_a^+})\,\tilde{V}_{a2}\,. \tag{2.6}$$

For simplicity, we have not explicitly written down the other weak contributions to ϵ_b, which can be found in ref. [17]. Here A_t is the trilinear coefficient, $\tilde{U}$ and $\tilde{V}$ are the two matrices (assumed to be real) that diagonalize the chargino mass matrix

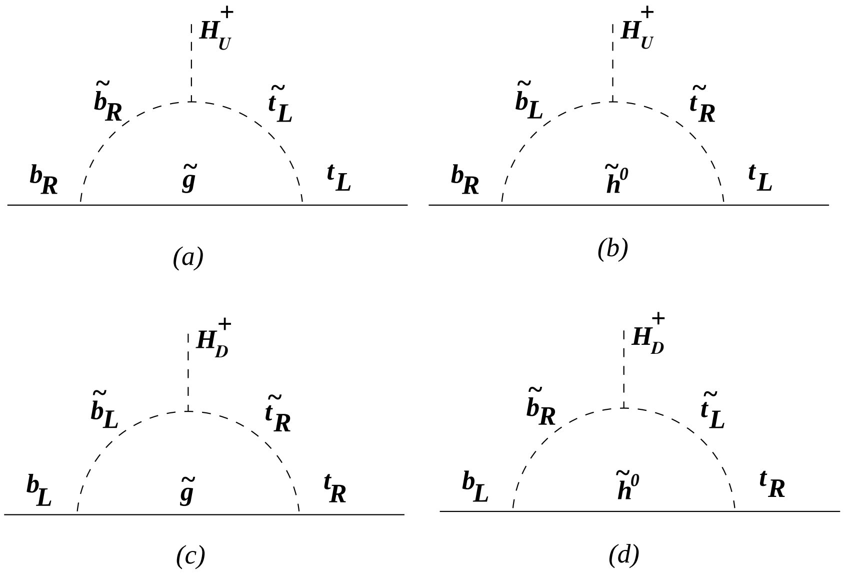

Figure 1: Feynman diagrams (for current squark eigenstates) representing the QCD (a) and Yukawa (b) contributions to $\epsilon'_b(t)$ and the QCD (c) and Yukawa (d) contribution to $\epsilon'_t(b)$.

according to

$$\tilde{U}\begin{pmatrix} M_2 & M_W\sqrt{2}\sin\beta \\ M_W\sqrt{2}\cos\beta & \mu \end{pmatrix}\tilde{V}^{-1} \tag{2.7}$$

and

$$H_2(x,y) = \frac{x\ln x}{(1-x)(x-y)} + \frac{y\ln y}{(1-y)(y-x)}\,. \tag{2.8}$$

Here and in the following we define, for generic indices α and β,

$$x_{\alpha\beta} \equiv \frac{m_\alpha^2}{m_\beta^2}\,. \tag{2.9}$$

The analogous contribution to the top quark mass (ϵ_t) is irrelevant for us, since it gives rise to terms suppressed by $\tan\beta$.

The effective Yukawa couplings of the charged Higgs current-eigenstates H_D^+ and H_U^+ (belonging to the doublets coupled to down- and up-type quarks, respectively) are given by

$$\mathcal{L} = \sum_d V_{td}\, y_t \bar{t}_R d_L \left[H_U^+ + \epsilon'_t(d)\, H_D^+\right] - \sum_u V_{ub}\, y_b \bar{u}_L b_R \left[H_D^+ + \epsilon'_b(u)\, H_U^+\right] + \text{h.c.} \tag{2.10}$$

The sum $\sum_u$ $(\sum_d)$ is over the three generation of up (down) type quarks and we have kept only the terms proportional to third-generation Yukawa couplings.

The $\epsilon'_{b,t}$ coefficients originate from the charged-current analogue of the diagrams leading to $\epsilon_{b,t}$ — see figure 1 — and they are given by

$$\begin{aligned}
\epsilon'_b(t) = & -\frac{2\,\alpha_s}{3\,\pi}\frac{\mu}{m_{\tilde g}}\Big[c_{\tilde t}^2 c_{\tilde b}^2\, H_2(x_{\tilde t_1\,\tilde g}, x_{\tilde b_2\,\tilde g}) + c_{\tilde t}^2 s_{\tilde b}^2\, H_2(x_{\tilde t_1\,\tilde g}, x_{\tilde b_1\,\tilde g}) + \\
& \qquad + s_{\tilde t}^2 c_{\tilde b}^2\, H_2(x_{\tilde t_2\,\tilde g}, x_{\tilde b_2\,\tilde g}) + s_{\tilde t}^2 s_{\tilde b}^2\, H_2(x_{\tilde t_2\,\tilde g}, x_{\tilde b_1\,\tilde g})\Big] - \\
& -\frac{y_t^2}{16\,\pi^2}\, N_{4a}\frac{A_t}{m_{\chi_a^0}}\Big[c_{\tilde t}^2 c_{\tilde b}^2\, H_2(x_{\tilde t_2\,\chi_a^0}, x_{\tilde b_1\,\chi_a^0}) + c_{\tilde t}^2 s_{\tilde b}^2\, H_2(x_{\tilde t_2\,\chi_a^0}, x_{\tilde b_2\,\chi_a^0}) + \\
& \qquad + s_{\tilde t}^2 c_{\tilde b}^2\, H_2(x_{\tilde t_1\,\chi_a^0}, x_{\tilde b_1\,\chi_a^0}) + s_{\tilde t}^2 s_{\tilde b}^2\, H_2(x_{\tilde t_1\,\chi_a^0}, x_{\tilde b_2\,\chi_a^0})\Big] N_{a3}^* \\
\epsilon'_t(b) = & -\frac{2\,\alpha_s}{3\,\pi}\frac{\mu}{m_{\tilde g}}\Big[c_{\tilde t}^2 c_{\tilde b}^2\, H_2(x_{\tilde t_2\,\tilde g}, x_{\tilde b_1\,\tilde g}) + c_{\tilde t}^2 s_{\tilde b}^2\, H_2(x_{\tilde t_2\,\tilde g}, x_{\tilde b_2\,\tilde g}) + \\
& \qquad + s_{\tilde t}^2 c_{\tilde b}^2\, H_2(x_{\tilde t_1\,\tilde g}, x_{\tilde b_1\,\tilde g}) + s_{\tilde t}^2 s_{\tilde b}^2\, H_2(x_{\tilde t_1\,\tilde g}, x_{\tilde b_2\,\tilde g})\Big] - \\
& -\frac{y_b^2}{16\,\pi^2}\, N_{4a}^*\frac{A_b}{m_{\chi_a^0}}\Big[c_{\tilde t}^2 c_{\tilde b}^2\, H_2(x_{\tilde t_1\,\chi_a^0}, x_{\tilde b_2\,\chi_a^0}) + c_{\tilde t}^2 s_{\tilde b}^2\, H_2(x_{\tilde t_1\,\chi_a^0}, x_{\tilde b_1\,\chi_a^0}) + \\
& \qquad + s_{\tilde t}^2 c_{\tilde b}^2\, H_2(x_{\tilde t_2\,\chi_a^0}, x_{\tilde b_2\,\chi_a^0}) + s_{\tilde t}^2 s_{\tilde b}^2\, H_2(x_{\tilde t_2\,\chi_a^0}, x_{\tilde b_1\,\chi_a^0})\Big] N_{a3}\,, \qquad (2.11)
\end{aligned}$$

where N is the matrix that diagonalizes the neutralino mass matrix, $c_{\tilde q} \equiv \cos\theta_{\tilde q}$, $s_{\tilde q} \equiv \sin\theta_{\tilde q}$, and the squarks eigenstates are $\tilde q_1 = c_{\tilde q}\,\tilde q_L + s_{\tilde q}\,\tilde q_R$ and $\tilde q_2 = -s_{\tilde q}\,\tilde q_L + c_{\tilde q}\,\tilde q_R$ have mass eigenvalues $m_{\tilde q_1} > m_{\tilde q_2}$. The quantity $\epsilon'_t(d)$ with $d \neq b$ is given by eq. (2.11) after setting $y_b = 0$, $\theta_{\tilde b} = 0$, and identifying $m_{\tilde b_{1,2}}$ with $m_{\tilde d_{1,2}}$. In the limit of exact weak SU(2) (i.e. $\tilde m_{b_L} = \tilde m_{t_L}$ and $\theta_{\tilde t} = \theta_{\tilde b} = 0$), the coefficients $\epsilon'_{b,t}$ coincide with $\epsilon_{b,t}$. However, their difference can be numerically significant, since we are interested also in cases of large stop mixing. Notice that the coefficients $\epsilon'_{b,t}$ (as well as $\epsilon_{b,t}$) are non-vanishing even in the limit in which all the supersymmetric masses are simultaneously sent to infinity. This means that the effective charged-Higgs theory [18] obtained by decoupling gluino and squarks does not correspond to what is usually called Model II. There are additional couplings, namely the ϵ and ϵ' coefficients in eqs. (2.5) and (2.10), which lead to $\tan\beta$-enhanced contributions to C_7 and C_8 at two loops. In the decoupling limit, $\epsilon'_{b,t} - \epsilon_{b,t}$ vanish, since they are proportional to SU(2)-breaking effects.

We can now express the interaction lagrangian in eq. (2.10) in terms of the would-be Goldstone boson $G^+ = \cos\beta H_D^+ + \sin\beta H_U^+$ and the physical charged Higgs $H^+ = -\sin\beta H_D^+ + \cos\beta H_U^+$. Replacing the Yukawa couplings y_t and y_b with the top and bottom quark masses, see eq. (2.5), we find

$$\begin{aligned}
\mathcal{L} = & \frac{g}{\sqrt{2}M_W}G^+\left\{\sum_d m_t\, V_{td}\bar t_R d_L - \sum_u m_b\, V_{ub}\frac{1+\epsilon'_b(u)\tan\beta}{1+\epsilon_b\tan\beta}\bar u_L b_R\right\} + \qquad (2.12) \\
& + \frac{g}{\sqrt{2}M_W}H^+\left\{\sum_d V_{td}\frac{m_t\,[1-\epsilon'_t(d)\tan\beta]}{\tan\beta}\bar t_R d_L + \sum_u V_{ub}\frac{m_b\tan\beta}{1+\epsilon_b\tan\beta}\bar u_L b_R\right\} + \text{h.c.}
\end{aligned}$$

From the interactions in eq. (2.12) we can now directly read the leading $\tan\beta$ higher-order contributions to the Wilson coefficients C_7 and C_8 in the case of a light charged Higgs, by recalling that the one-loop diagram is proportional to the product of the Higgs couplings to the $\bar{t}_L b_R$ and $\bar{s}_L t_R$ currents,

$$C_{7,8}^{(SM)}(\text{leading} \tan\beta) = \frac{[\epsilon_b - \epsilon_b'(t)] \tan\beta}{1+\epsilon_b \tan\beta} F_{7,8}^{(2)}(x_t) \tag{2.13}$$

$$C_{7,8}^{(H^\pm)}(\text{leading} \tan\beta) = -\frac{[\epsilon_t'(s) + \epsilon_b] \tan\beta}{1+\epsilon_b \tan\beta} F_{7,8}^{(2)}(y_t)\,. \tag{2.14}$$

In the limit of very heavy supersymmetric particles, the $\tan\beta$-enhanced terms in $C_{7,8}^{(SM)}$ vanish, consistently with the theorem of decoupling. However, for finite supersymmetric masses, we find $\tan\beta$-enhanced corrections to the Standard Model contribution.

The $\tan\beta$-enhanced terms in the charged-Higgs contribution do not vanish in the decoupling limit because, as previously explained, additional couplings to those of the charged-Higgs Model II are recovered in this limit. The term proportional to ϵ_b (previously discussed in ref. [15]) originates from the modified relation between the bottom mass and Yukawa coupling in eq. (2.5), while the term proportional to ϵ_t' comes from the modified charged Higgs boson vertex in eq. (2.12) [11].

Notice that $\tan\beta$ enhanced terms in the charged-Higgs contribution to $B \to X_s\gamma$ can be induced not only by vertex corrections, but also by corrections to the charged-Higgs propagator. They arise from loop corrections to the propagator $\langle H_U^+ H_D^- \rangle$ which vanishes at tree level, in the limit $\tan\beta \to \infty$. In this limit, the Higgs potential classically has a Peccei-Quinn (PQ) symmetry (since the scalar Higgs mixing mass parameter B_μ has to vanish [16]) and therefore any contribution to $\langle H_U^+ H_D^- \rangle$ must be proportional to μ, which is the only surviving PQ-breaking parameter. This propagator receives contributions from higgsino-gaugino loops or stop loops. However, the loop contributions which renormalize B_μ are simply reabsorbed in the definition of $\tan\beta$, which is treated here as an input parameter. Analogously, the effect of PQ-violating quartic Higgs couplings (of the kind $H_D H_U H_U^\dagger H_U$) obtained by integrating out heavy supersymmetric particles (and which survive as renormalizable interactions in the decoupling limit) are absorbed in the redefinition of the vacuum expectation values. Nevertheless, supersymmetric particles with masses comparable to M_H can lead to momentum-dependent contributions to $\langle H_U^+ H_D^- \rangle$, which generate potentially significant $\tan\beta$ enhanced effects in $C_{7,8}$. In particular, this could be the case for the higgsino-gaugino loop, if charginos are light, although the effect is suppressed by two powers of the $\mathrm{SU}(2)_W$ breaking scale and four powers of the weak gauge coupling constant. For this reason, we will neglect this effect here.

3. Chargino contributions

At the one-loop level, the leading contribution to $C_{7,8}(\mu_W)$ in the large $\tan\beta$ limit from chargino and squark exchange is given by

$$C^{\chi}_{7,8}(\mu_W) = \frac{1}{\cos\beta}\sum_{a=1,2}\left\{\frac{\tilde{U}_{a2}\tilde{V}_{a1}M_W}{\sqrt{2}m_{\chi_a^+}}\left[F^{(3)}_{7,8}(x_{\tilde{q}\,\chi_a^+}) - c_{\tilde{t}}^2\,F^{(3)}_{7,8}(x_{\tilde{t}_1\,\chi_a^+}) - s_{\tilde{t}}^2\,F^{(3)}_{7,8}(x_{\tilde{t}_2\,\chi_a^+})\right] + \right.$$
$$\left. + \, s_{\tilde{t}}\,c_{\tilde{t}}\frac{\tilde{U}_{a2}\,\tilde{V}_{a2}\,\bar{m}_t}{2\sin\beta\,m_{\chi_a^+}}\left[F^{(3)}_{7,8}(x_{\tilde{t}_1\,\chi_a^+}) - F^{(3)}_{7,8}(x_{\tilde{t}_2\,\chi_a^+})\right]\right\}, \tag{3.1}$$

where $m_{\tilde{q}}$ is a common mass for the first two generation squarks, and

$$F^{(3)}_7(x) = \frac{5-7x}{6(x-1)^2} + \frac{x(3x-2)}{3(x-1)^3}\ln x\,, \tag{3.2}$$
$$F^{(3)}_8(x) = \frac{1+x}{2(x-1)^2} - \frac{x}{(x-1)^3}\ln x\,. \tag{3.3}$$

Eq. (3.1) contains two terms which grow linearly with $\tan\beta$, in the large $\tan\beta$ limit. The first line originates from a chargino-stop loop, in which the $\langle H_U\rangle$ insertion (signalling the $\tan\beta$ enhancement) occurs in the gaugino-higgsino mixing. The second one, instead, comes from a diagram mediated by a higgsino-stop loop and the $\langle H_U\rangle$ insertion occurs in the stop left-right mixing. The first term suffers from a suppression with respect to the second one of a g^2/y_t^2 factor (because of the gaugino coupling) and of a squark GIM factor. However, in many cases of interest in which the stop mass is significantly split from the $\tilde{c}_L$ and $\tilde{u}_L$ masses, the two diagram contributions are comparable in size.

The only terms in the chargino contributions enhanced by an extra $\tan\beta$ factor at the two-loop order originate from the ϵ_b coefficient, i.e. from the modified relation between m_b and y_b in eq. (2.5). Therefore the leading higher-order terms are taken into account by dividing the one-loop expression of (20) by $(1+\epsilon_b\tan\beta)$ (see the first paper of [15]). Since the one-loop contribution is linear in $\tan\beta$, the two-loop term is $\mathcal{O}(\tan^2\beta)$.

This method of extracting the leading $\tan\beta$ terms cannot give us the two-loop $\mathcal{O}(\tan\beta)$ chargino contribution, which requires a full diagrammatic calculation, outside the scope of this work. In the present analysis we are going to keep only the subleading terms in $\tan\beta$ which can be potentially large. They are related to three different effects. (i) Two-loop $\mathcal{O}(\tan\beta)$ terms which are power-suppressed only by the charged-Higgs mass and not by any supersymmetric particle heavy mass. These can be important in models in which the charged-Higgs is significantly lighter than gluino and squarks, and have been computed in the previous section. (ii) Two-loop terms with large logarithms of the ratio $\mu_{\rm SUSY}/\mu_W$ related to the different renormalizations of Yukawa couplings in the Higgs/higgsino vertices.

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(iii) Two-loop terms enhanced by $\ln(\mu_{\rm SUSY}/\mu_W)$ connected with the anomalous dimensions of the magnetic and chromo-magnetic effective operators. The last two classes of terms become important when the scale of the supersymmetric colored particles is significantly higher than the W and top masses. The effect is particularly sizeable because the anomalous dimensions of the relevant operators are quite large.

Let us now discuss the terms in class (ii). The Yukawa coupling $\tilde{y}_t$ appearing in the chargino-stop-sbottom vertex is related to the ordinary top-quark Yukawa coupling y_t only by supersymmetry. Therefore at the scale of the heavy supersymmetric particle masses $\mu_{\rm SUSY}$, we have $\tilde{y}_t(\mu_{\rm SUSY}) = y_t(\mu_{\rm SUSY})$. After decoupling the supersymmetric modes, $\tilde{y}_t$ is frozen, while y_t evolves according to the Standard Model renormalization group. Large logarithms are generated when we express $\tilde{y}_t$ in terms of y_t or, in other words, in terms of the top quark mass evaluated at the weak scale. The resummation of these logarithms gives

$$\begin{aligned}\tilde{y}_t(\mu_{\rm SUSY}) = y_t(\mu_W) \left[\frac{\alpha_s(\mu_{\rm SUSY})}{\alpha_s(m_t)}\right]^{4/7} \left[\frac{\alpha_s(m_t)}{\alpha_s(\mu_W)}\right]^{12/23} \times \\ \times \frac{1}{\sqrt{1+\frac{9y_t^2(m_t)}{8\pi\alpha_s(m_t)}\left\{\left[\frac{\alpha_s(\mu_{\rm SUSY})}{\alpha_s(m_t)}\right]^{1/7}-1\right\}}}\end{aligned} \tag{3.4}$$

Here we have used six active quark flavors above the the top quark threshold which is generally higher than μ_W. In practice, however, $m_t = O(\mu_W)$ and the modification in the running between m_t and μ_W is numerically small. At worst, for $\mu_W = 40\,\mathrm{GeV}$ it leads to a 1% modification in the Yukawa coupling. Therefore, the logarithms in class (ii) are taken into account by using $m_t(\mu_{\rm SUSY})$ in the chargino contribution.

The logarithms in class (iii) are taken into account by considering the evolution of the effective operators from the scale $\mu_{\rm SUSY}$ to the scale μ_W (notice that no new operator is involved)[1]

$$\begin{aligned}C_7^\chi(\mu_W) &= \eta^{-16/3\beta_0}\, C_7^\chi(\mu_{\rm SUSY}) + \frac{8}{3}\left(\eta^{-14/3\beta_0} - \eta^{-16/3\beta_0}\right) C_8^\chi(\mu_{\rm SUSY}) \\ C_8^\chi(\mu_W) &= \eta^{-14/3\beta_0}\, C_8^\chi(\mu_{\rm SUSY})\end{aligned} \tag{3.5}$$

where $\eta \equiv \alpha_s(\mu_{\rm SUSY})/\alpha_s(\mu_W) = [1-(\beta_0/2\pi)\ln(\mu_{\rm SUSY}/\mu_W)]^{-1}$ and $\beta_0 = -7$ corresponding to six active flavors.

[1] While the running of the Wilson coefficients from μ_W to μ_b is performed at the next-to-leading-log level, we keep here only the leading logs of $\mu_{\rm SUSY}/\mu_W$. This should be sufficient because of the high scale at which the running takes place. The resummation of next-to-leading logs would require the complete $O(\alpha_s)$ matching conditions at $\mu_{\rm SUSY}$.

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We can judge the effect of the resummation by retaining only the first logarithm in eqs. (3.5)

$$
\begin{aligned}
\delta C_7^\chi(\mu_W) &= -\frac{4\alpha_s(\mu_W)}{3\pi}\left[C_7^\chi(\mu_{\rm SUSY}) - \frac{1}{3}C_8^\chi(\mu_{\rm SUSY})\right]\ln\frac{\mu_{\rm SUSY}^2}{\mu_W^2} \\
\delta C_8^\chi(\mu_W) &= -\frac{7\alpha_s(\mu_W)}{6\pi}C_8^\chi(\mu_{\rm SUSY})\ln\frac{\mu_{\rm SUSY}^2}{\mu_W^2}\,.
\end{aligned}
\tag{3.6}
$$

For instance, taking $C_8^\chi(\mu_{\rm SUSY}) = 0$ and $\mu_{\rm SUSY} = 1\,\mathrm{TeV}$, the not-resummed evolutions of C_7^χ from $\mu_{\rm SUSY}$ to μ_W is proportional to

$$
-\frac{4\alpha_s(\mu_W)}{3\pi}\ln\frac{\mu_{\rm SUSY}^2}{\mu_W^2} = -0.257\,, \qquad -\frac{4\alpha_s(\mu_{\rm SUSY})}{3\pi}\ln\frac{\mu_{\rm SUSY}^2}{\mu_W^2} = -0.191\,, \tag{3.7}
$$

while the resummed expression gives

$$
\eta^{16/21} - 1 = -0.203\,. \tag{3.8}
$$

This demonstrates that the choice of evaluating α_s at $\mu_{\rm SUSY}$ can incorporate most of the effect of the resummation.

There is another important case in which we can retain at next-to-leading order all terms linear in $\tan\beta$ not suppressed by heavy masses. This is when the charginos and a mostly right-handed stop are significantly lighter than the gluino and the other squarks. This scenario was discussed in ref. [11], where the leading terms in an expansion in the ratio of light to heavy supersymmetric particle masses were derived. Also in that situation large logs (non-decoupling logs) dominate the next-to-leading corrections and may lead to sizeable effects in the branching ratio. It is not difficult to resum these logarithms. The crucial point is that, in the effective theory where only the gluino and heavy squarks have been integrated out at $\mu_{\rm SUSY}$, the gaugino and higgsino couplings renormalize differently from the ordinary gauge and Yukawa couplings. As a result, the couplings in the chargino interactions at the electroweak scale differ from the Standard Model couplings by $O(\alpha_s)$ contributions enhanced by large logarithms. If we denote by $\tilde{y}$ and $\tilde{g}$ the Yukawa and weak gauge couplings in the chargino vertex, supersymmetry requires $\tilde{y}(\mu_{\rm SUSY}) = y(\mu_{\rm SUSY})$ and $\tilde{g}(\mu_{\rm SUSY}) = g(\mu_{\rm SUSY})$. Since we evaluate the diagram involving charginos and light stop at the scale μ_W, we have to express the supersymmetric couplings at the scale μ_W in terms of Standard Model couplings. Using the QCD renormalization group equations, we obtain

$$
\tilde{g}(\mu_W) = g(\mu_W)\eta^{2/\beta_0}\,, \qquad \tilde{y}_t(\mu_W) = y_t(\mu_{\rm SUSY})\eta^{2/\beta_0}\,, \qquad \tilde{y}_b(\mu_W) = y_b(\mu_W)\eta^{-2/\beta_0}\,, \tag{3.9}
$$

where η should be evaluated with $\beta_0 = -41/6$, the beta-function coefficient including 6 quark flavours and one squark. For convenience we have related $\tilde{y}_t(\mu_W)$ to the top Yukawa coupling at $\mu_{\rm SUSY}$, but $\tilde{y}_b(\mu_W)$ is expressed in terms of $y_b(\mu_W)$ in order to reconstruct the operators Q_7 and Q_8 at the proper scale. The resummation of the QCD logarithms is therefore implemented by evaluating m_t at the scale $\mu_{\rm SUSY}$ and multiplying by η^{4/β_0} the functions $F^{(1)}_{7,8}$ (which describe diagrams proportional to two powers of $\tilde{g}(\mu_W)$ or $\tilde{y}_t(\mu_W)$). On the other hand, the functions $F^{(3)}_{7,8}$ should not be rescaled, because they arise from diagrams proportional to $\tilde{y}_b(\mu_W)\tilde{y}_t(\mu_W)$ or $\tilde{y}_b(\mu_W)\tilde{g}(\mu_W)$. We will present the final result in the next section.

The large logarithms proportional to Yukawa couplings can also be resummed with the same approach we have discussed here. However, in this case it is crucial to maintain the electroweak gauge invariance of the effective theory between the scales $\mu_{\rm SUSY}$ and μ_W, and therefore the resummation method is valid only in the limit of pure right-handed light stop.

4. Summary of the leading higher-order contributions

We now summarize the formulae for the supersymmetric contribution to the Wilson coefficients that contain the leading higher-order effects in the scenario in which the colored supersymmetric particles have mass $\mathcal{O}(\mu_{\rm SUSY} \sim 1\,{\rm TeV})$ with the possibility that the physical charged Higgs and the charginos could be lighter with masses $\mathcal{O}(\mu_W)$. The expressions, up to next-to-leading order, for the SM and charged Higgs contribution to $C_{7,8}$ can be found in ref. [4]. The next-to-leading order charged higgs contribution contains terms enhanced by potentially large logarithms of the kind $\ln(m_H/\mu_W)$ which we have not resummed (see ref. [19] for a complete resummation). The chargino contribution is instead given by eqs. (3.5) with

$$
\begin{aligned}
C^{\chi}_{7,8}(\mu_{\rm SUSY}) = \sum_{a=1,2} \Bigg\{ & \frac{2}{3}\frac{M_W^2}{\tilde{m}^2}\tilde{V}_{a1}^2 F^{(1)}_{7,8}(x_{\tilde{q}\chi^+_a}) - \\
& - \frac{2}{3}\left(c_{\tilde{t}}\tilde{V}_{a1} - s_{\tilde{t}}\tilde{V}_{a2}\frac{\bar{m}_t(\mu_{\rm SUSY})}{\sqrt{2}\sin\beta\, M_W}\right)^2 \frac{M_W^2}{m^2_{\tilde{t}_1}} F^{(1)}_{7,8}(x_{\tilde{t}_1\chi^+_a}) - \\
& - \frac{2}{3}\left(s_{\tilde{t}}\tilde{V}_{a1} + c_{\tilde{t}}\tilde{V}_{a2}\frac{\bar{m}_t(\mu_{\rm SUSY})}{\sqrt{2}\sin\beta\, M_W}\right)^2 \frac{M_W^2}{m^2_{\tilde{t}_2}} F^{(1)}_{7,8}(x_{\tilde{t}_2\chi^+_a}) + \\
& + \frac{K}{\cos\beta}\Bigg(\frac{\tilde{U}_{a2}\tilde{V}_{a1}M_W}{\sqrt{2}m_{\chi^+_a}}\Big[F^{(3)}_{7,8}(x_{\tilde{q}\chi^+_a}) - \\
& \qquad - c^2_{\tilde{t}}F^{(3)}_{7,8}(x_{\tilde{t}_1\chi^+_a}) - s^2_{\tilde{t}}F^{(3)}_{7,8}(x_{\tilde{t}_2\chi^+_a})\Big] + \\
& + s_{\tilde{t}}\frac{\tilde{U}_{a2}\tilde{V}_{a2}\bar{m}_t(\mu_{\rm SUSY})}{2\sin\beta\, m_{\chi^+_a}}\Big[F^{(3)}_{7,8}(x_{\tilde{t}_1\chi^+_a}) - F^{(3)}_{7,8}(x_{\tilde{t}_2\chi^+_a})\Big]\Bigg)\Bigg\}.
\end{aligned}
\tag{4.1}
$$

In eq. (4.1) $\bar{m}_t(\mu_{\rm SUSY})$ is expressed in terms of the top quark Yukawa coupling given by eq. (3.4) and the K factor can be taken equal to 1 for small values of $\tan\beta$, while in the large $\tan\beta$ scenario is given by $K = 1/(1+\epsilon_b\tan\beta)$. In the latter case one

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has to keep the contribution given by eq. (2.13) and, if the physical charged Higgs is assumed to be light, one can also take into account the dominant contribution to the terms enhanced by a single power of $\tan\beta$ that is given by eq. (2.14).

We now consider the second scenario in which charginos and the mostly right-handed stop are significantly lighter than the gluino and the other squarks. In this situation the NLO Wilson coefficients contain large logarithms of the ratio of light to heavy mass that were not resummed in ref. [11], but can be easily included to all orders using the results of the previous section.

We identify $\mu_{\rm SUSY}$ with the gluino mass scale, $m_{\tilde g}$, and μ_W with the charginos-light stop scale and rewrite the supersymmetric contribution to the Wilson coefficient at the weak scale as

$$\begin{aligned}
C_7^{\chi}(\mu_W) = \sum_{a=1,2} \eta^{-\frac{16}{3\beta_0}} \Bigg\{ & \frac{2}{3}\frac{M_W^2}{\tilde m^2}\tilde V_{a1}^2 F_7^{(1)}(x_{\tilde q\,\chi_a^+}) - \\
& - \frac{2}{3}\left(c_{\tilde t}\,\tilde V_{a1} - s_{\tilde t}\,\tilde V_{a2}\frac{\bar m_t(\mu_{\rm SUSY})}{\sqrt{2}\sin\beta\, M_W}\right)^2 \frac{M_W^2}{m_{\tilde t_1}^2} F_7^{(1)}(x_{\tilde t_1\,\chi_a^+}) + \\
& + \frac{K}{\cos\beta}\left(\frac{\tilde U_{a2}\tilde V_{a1} M_W}{\sqrt{2} m_{\chi_a^+}}\left[F_7^{(3)}(x_{\tilde q\,\chi_a^+}) - c_{\tilde t}^2\, F_7^{(3)}(x_{\tilde t_1\,\chi_a^+})\right] + \right. \\
& \left. + s_{\tilde t}\, c_{\tilde t}\frac{\tilde U_{a2}\,\tilde V_{a2}\,\bar m_t(\mu_{\rm SUSY})}{2\sin\beta\, m_{\chi_a^+}} F_7^{(3)}(x_{\tilde t_1\,\chi_a^+})\right)\Bigg\} + \\
+ \sum_{a=1,2} \frac{8}{3}\left(\eta^{-\frac{14}{3\beta_0}} - \eta^{-\frac{16}{3\beta_0}}\right)\{7\to 8\} + & \\
+ \sum_{a=1,2} \Bigg\{ & -\frac{2}{3}\eta^{\frac{4}{\beta_0}}\left(s_{\tilde t}\,\tilde V_{a1} + c_{\tilde t}\,\tilde V_{a2}\frac{\bar m_t(\mu_{\rm SUSY})}{\sqrt{2}\sin\beta\, M_W}\right)^2 \frac{M_W^2}{m_{\tilde t_2}^2} F_7^{(1)}(x_{\tilde t_2\,\chi_a^+}) - \\
& - \frac{1}{\cos\beta}\left[\frac{\tilde U_{a2}\tilde V_{a1} M_W}{\sqrt{2} m_{\chi_a^+}} s_{\tilde t}^2\,(K + \epsilon_b\tan\beta) + s_{\tilde t}\, c_{\tilde t}\frac{\tilde U_{a2}\,\tilde V_{a2}\,\bar m_t(\mu_{\rm SUSY})}{2\sin\beta\, m_{\chi_a^+}} \times \right. \\
& \left. \times\left(K + \epsilon_b\tan\beta\frac{\bar m_t(\mu_W)}{\bar m_t(\mu_{\rm SUSY})}\right)\right] F_7^{(3)}(x_{\tilde t_2\,\chi_a^+})\Bigg\} + \\
+ \frac{\alpha_s(\mu_W)}{4\pi}\delta^S C_7^{(1)}(\mu_W)\,. &
\end{aligned} \tag{4.2}$$

In eq. (4.2) the term $\{7\to 8\}$ is the same as the one in the previous curly bracket, with the F_7 replaced by the F_8 functions. Also, $\beta_0 = -41/6$ and the term $\delta^S C_7^{(1)}(\mu_W)$ is given in ref. [11, eq. (12)][2] with the following modifications to adjust for the resummation: i) the $\ln m_{\tilde g}^2/m_{\chi_j}^2$ in the functions $G_{7,8}^{\chi_j,1}$ in [11, eqs. (15)–(17)] must be replaced by $\ln \mu_W^2/m_{\chi_j}^2$; ii) the $\ln \mu_W^2/m_{\tilde g}^2$ in the functions R_i in [11, eq. (19)] must be dropped.

[2] In the published version of ref. [11] there are typos corrected in the hep-ph archive version.

The expression for the $C_8^\chi(\mu_W)$ coefficient can be obtained from eq. (4.2) by dropping the $\{7 \to 8\}$ term, by replacing the $\eta^{-16/3\beta_0}$ factor that multiplies the first curly bracket with $\eta^{-14/3\beta_0}$, and by substituting all the F_7 with the F_8 functions.

5. Numerical analysis

In this section we briefly illustrate the impact of the improved formulae, collected in the previous section, in the calculation of $B \to X_s\gamma$ branching ratio. We will consider here the case of a minimal supergravity model, in which gaugino and squark masses have common values $m_{1/2}$ and m_0, respectively, at the GUT scale. In the calculation of the branching ratio for $B \to X_s\gamma$ we include all known perturbative and non-perturbative effects. Next-to-leading order gluonic corrections to the Wilson coefficients at the electroweak scale are included for the Standard Model [3, 4] and charged Higgs contributions [4, 9], together with the $\tan\beta$ enhanced terms introduced in section 2. For what concerns the chargino contributions, only the leading logs — in the resummed form — and the $\tan\beta$ enhanced terms discussed in section 3 have been included beyond leading order. We estimate that residual uncalculated next-to-leading order contributions, which do not present in this scenario any obvious enhancement, should be smaller than a few percent. In the following we neglect $\tan\beta$ enhanced terms of electroweak origin.

In figures 2 and 3 we show the branching ratio for $B \to X_s\gamma$, as a function of $\tan\beta$, with the parameter choice $m_0 = 600\,\text{GeV}$, $m_{1/2} = 400\,\text{GeV}$, and the common

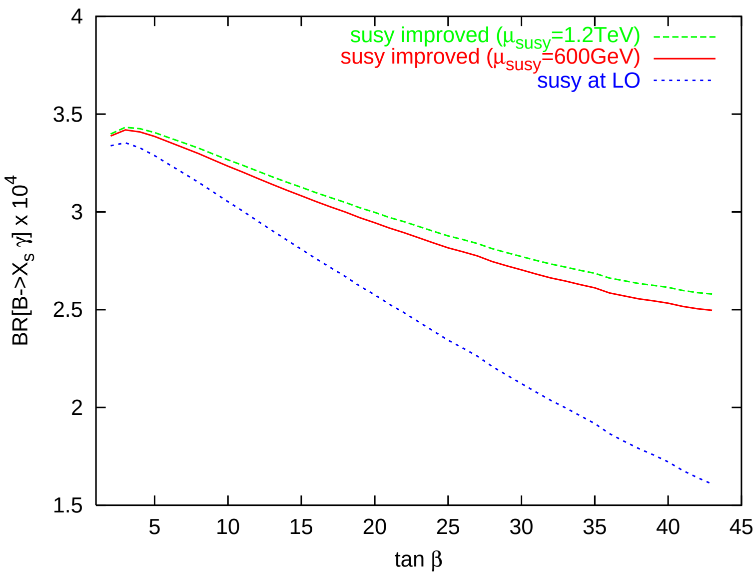

Figure 2: Branching ratio for $B \to X_s\gamma$ in a minimal supergravity scenario with $m_0 = 600\,\text{GeV}$, $m_{1/2} = 400\,\text{GeV}$, $A_0 = 0$, and $\mu > 0$ as a function of $\tan\beta$. The solid and dashed lines represent our improved framework for $\mu_{\text{SUSY}} = 600\,\text{GeV}$ and $1.2\,\text{TeV}$, while the dotted line represents the results of the calculation with LO supersymmetric contributions.

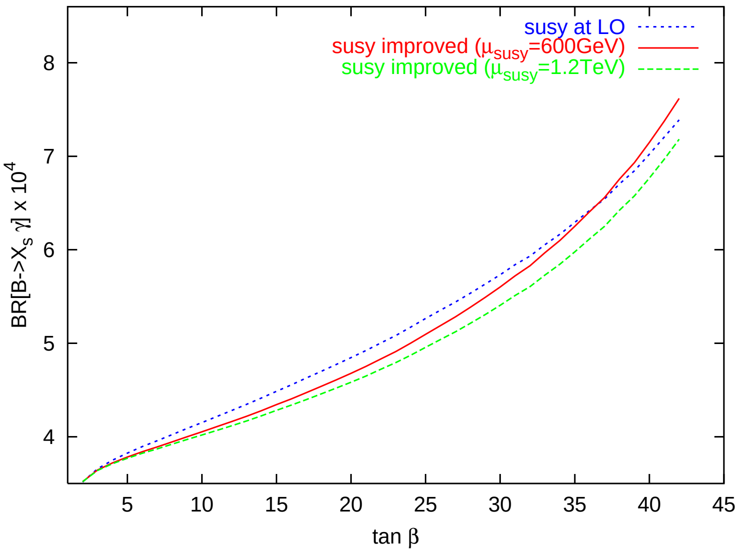

Figure 3: Same as figure 2 but with $\mu < 0$.

trilinear coupling at the GUT scale $A_0 = 0$. The two figures correspond to the two possible signs of μ, with $|\mu|$ determined by the electroweak breaking condition. This scenario is characterized by squark, charged higgs and gluino masses clustered between 700 GeV and 1 TeV, with the charginos somewhat lighter. Therefore we conservatively vary $\mu_{\rm SUSY}$ between the two edges of the physical heavy-mass spectrum, and plot the two extreme cases $\mu_{\rm SUSY} = 600\,\text{GeV}$ (solid line) and 1.2 TeV (dashed line). The dependence of the results on $\mu_{\rm SUSY}$ appears to be quite mild. The improved predictions are then compared with the same calculation with supersymmetric contributions to the Wilson coefficients implemented at μ_W without improvements. The case of $\mu > 0$ is characterized by destructive interference between Standard Model and chargino contributions. In this situation the improvements have a significant effect. For $\tan\beta = 40$ there is a 50% enhancement of the branching ratio. In the case of negative μ, there is constructive interference and the impact of the improved corrections is more limited.

We have studied the scale ambiguity of the results and found very small dependence on the matching scale μ_W. The dependence on the μ_b scale is unchanged with respect to previous analysis [4] and of the order of a few percent. Finally, the dependence on the choice of $\mu_{\rm SUSY}$ is shown in the plots and amounts at most to 3% (6%) for positive (negative) μ in the branching fraction, for very large $\tan\beta$. An estimate of the residual theoretical uncertainty can be also obtained by noting that in the context of the next-to-leading calculation our improvements can be implemented in two possible ways which differ by higher order effects only. One possibility is to use (3.9) in (3.5) and then identify the results as improved leading order coefficients at μ_W. The other option is to identify the difference between the results of (3.5) and the leading order coefficients as the $O(\alpha_s)$ or next-to-leading correction to the Wilson coefficient at μ_W. The difference between the two procedures is also $O(5\%)$.

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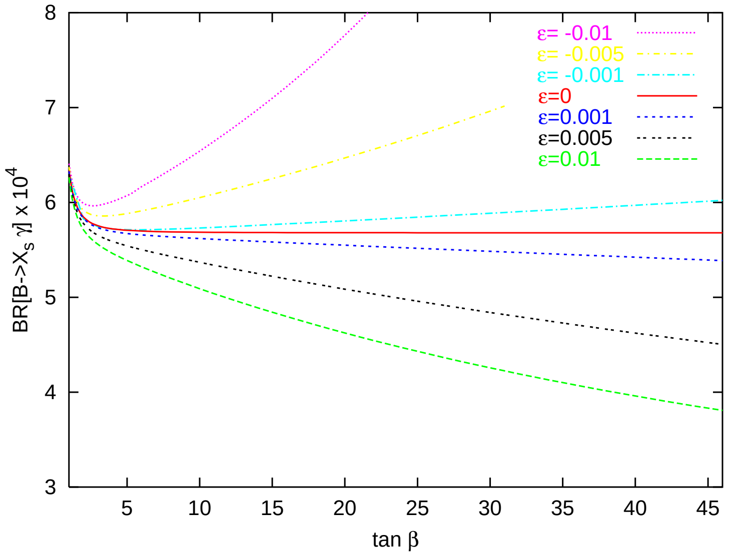

Figure 4: Branching ratio for $B \to X_s\gamma$ in a two-Higgs doublet model with the charged-Higgs mass $M_H = 150\,\mathrm{GeV}$, for different values of $\epsilon \equiv \epsilon_{b,t} = \epsilon'_{b,t}$.

We also want to comment on an analysis which has appeared in ref. [20]. The authors use the next-to-leading calculation of ref. [11], which assumes the hierarchy $m_{\tilde{t}_1} \gg m_{\tilde{t}_2}$, to compute the branching ratio for $B \to X_s\gamma$ for the same scenario considered by us in figures 2 and 3. Unfortunately, the mass hierarchy assumed in ref. [11] is not satisfied here, and the approximate formulae miss the cancellation between the contributions from the two stop mass eigenstates. In this situation the misuse of the result of ref. [11] may lead to artificially large effects. Indeed, as shown in figures 2 and 3, our analysis does not confirm the large enhancement claimed in ref. [20].

Finally, let us consider the scenario in which the supersymmetric particles are very heavy and let us focus on the contribution from the charged Higgs. As explained in section 2, the existence of the couplings $\epsilon_{b,t}$ and $\epsilon'_{b,t}$ modifies the predictions of what is generally called two-Higgs doublet model II even in the decoupling limit. The impact of the next-to-leading order calculation is illustrated in figure 4. This figure shows the branching ratio for $B \to X_s\gamma$ as a function of $\tan\beta$ for $M_H = 150\,\mathrm{GeV}$. We have chosen $\epsilon = \epsilon_{b,t} = \epsilon'_{b,t}$, and varied its numerical value. For $\epsilon > 0$ the next-to-leading corrections reduce the leading order result. The effect at large $\tan\beta$ can be very significant and it allows to consider values of the charged-Higgs mass, which were previously considered excluded, unless charginos and stops were comparably light. The impact of the ϵ corrections on the charged-Higgs mass lower limit from $B \to X_s\gamma$ is quantified in figure 5. This figure is obtained by combining in quadrature theoretical and experimental errors and using the most recent experimental measurements for $B \to X_s\gamma$ [1]. Notice that for $\tan\beta = 20$ values of the charged-Higgs mass as low as 150 GeV are allowed for $\epsilon = 10^{-2}$.

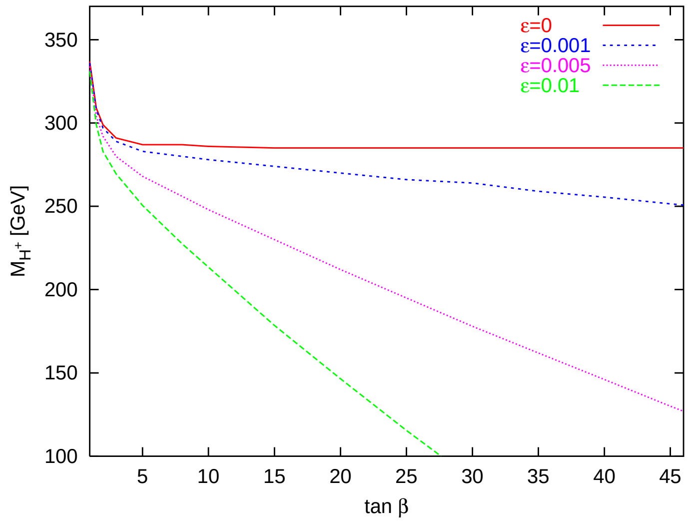

Figure 5: Lower bounds on the charged Higgs boson mass obtained from the experimental measurement of the branching ratio for $B \to X_s\gamma$ in a two-Higgs doublet model, for different values of $\epsilon \equiv \epsilon_{b,t} = \epsilon'_{b,t}$ and as a function of $\tan\beta$.

Acknowledgments

We are grateful to G. Ganis and M. Misiak for useful discussions and to R. Rattazzi for very interesting and important remarks.

References

[1] CLEO collaboration, *Improved Measurement of* $B(b \to s\gamma)$, CLEO CONF 98-17, ICHEP98-1011; ALEPH collaboration, *A measurement of the inclusive* $b \to s\gamma$ *branching ratio*, *Phys. Lett.* **B 429** (1998) 169;
M. Nakao, for the Belle collaboration, *Studies of radiative B-meson decays with* Belle, talk at ICHEP 2000, Osaka, July 2000.

[2] K. Chetyrkin, M. Misiak and M. Munz, *Weak radiative B-meson decay beyond leading logarithms*, *Phys. Lett.* **B 400** (1997) 206 [hep-ph/9612313], erratum *ibid.* **B425** (1998) 414.

[3] K. Adel and Y.-P. Yao, *Exact* α_s *calculation of* $b \to s + \gamma$, $b \to s + g$, *Phys. Rev.* **D 49** (1994) 4945 [hep-ph/9308349];
C. Greub and T. Hurth, *Two-loop matching of the dipole operators for* $b \to s\gamma$ *and* $b \to s$ *gluon*, *Phys. Rev.* **D 56** (1997) 2934 [hep-ph/9703349];
A.J. Buras, A. Kwiatkowski and N. Pott, *Next-to-leading order matching for the magnetic photon penguin operator in the* $b \to X_s\gamma$ *decay*, *Nucl. Phys.* **B 517** (1998) 353 [hep-ph/9710336].

[4] M. Ciuchini, G. Degrassi, P. Gambino and G.F. Giudice, *Next-to-leading QCD corrections to $b \to X_s\gamma$: standard model and two-higgs doublet model*, *Nucl. Phys.* **B 527** (1998) 21 [hep-ph/9710335].

[5] A. Ali and C. Greub, *Inclusive photon energy spectrum in rare b decays*, *Z. Physik* **C 49** (1991) 431; *A profile of the final states in $B \to X_s\gamma$ and an estimate of the branching ratio $BR(B \to K * \gamma)$*, *Phys. Lett.* **B 259** (1991) 182; *A determination of the ckm matrix element ratio $|V_{ts}|/|V_{cb}|$ from the rare B Decays $B \to K * +\gamma$ and $B \to X_s + \gamma$*, *Z. Physik* **C 60** (1993) 433; *Photon energy spectrum in $B \to X_s + \gamma$ and comparison with data*, *Phys. Lett.* **B 361** (1995) 146 [hep-ph/9506374];
C. Greub, T. Hurth and D. Wyler, *Virtual corrections to the decay $B \to s\gamma$*, *Phys. Lett.* **B 380** (1996) 385 [hep-ph/9602281]; *Virtual $o(\alpha_s)$ corrections to the inclusive decay $b \to s\gamma$*, *Phys. Rev.* **D 54** (1996) 3350 [hep-ph/9603404];
N. Pott, *Bremsstrahlung corrections to the decay $b \to s\gamma$*, *Phys. Rev.* **D 54** (1996) 938 [hep-ph/9512252].

[6] P. Gambino and U. Haisch, *Electroweak effects in radiative B decays*, *J. High Energy Phys.* **09** (2000) 001 [hep-ph/0007259].

[7] M.B. Voloshin, *Large $O(m_c^{-2})$ nonperturbative correction to the inclusive rate of the decay $B \to X_s\gamma$*, *Phys. Lett.* **B 397** (1997) 295 [hep-ph/9612483];
A. Khodjamirian, R. Ruckl, G. Stoll and D. Wyler, *QCD estimate of the long-distance effect in $B \to K * \gamma$*, *Phys. Lett.* **B 402** (1997) 167 [hep-ph/9702318];
Z. Ligeti, L. Randall and M.B. Wise, *Comment on nonperturbative effects in $\bar{B} \to X_s\gamma$*, *Phys. Lett.* **B 402** (1997) 178 [hep-ph/9702322];
A.K. Grant, A.G. Morgan, S. Nussinov and R.D. Peccei, *Comment on nonperturbative $O(1/m_c^2)$ corrections to $\gamma(\bar{B} \to X_s\gamma)$*, *Phys. Rev.* **D 56** (1997) 3151 [hep-ph/9702380];
G. Buchalla, G. Isidori and S.J. Rey, *Corrections of order Λ^2_{QCD}/m_c^2 to inclusive rare B decays*, *Nucl. Phys.* **B 511** (1998) 594 [hep-ph/9705253];
A.F. Falk, M. Luke and M.J. Savage, *Nonperturbative contributions to the inclusive rare decays $B \to X_s\gamma$ and $B \to X_s\ell^+\ell^-$*, *Phys. Rev.* **D 49** (1994) 3367 [hep-ph/9308288].

[8] A. Czarnecki and W.J. Marciano, *Electroweak radiative corrections to $b \to s\gamma$*, Phys. Rev. Lett. **81** (191998) 277 [hep-ph/9804252];
A.L. Kagan and M. Neubert, *QCD anatomy of $B \to X_s\gamma$ decays*, *Eur. Phys. J.* **C 7** (1999) 5 [hep-ph/9805303].

[9] F.M. Borzumati and C. Greub, *2HDMs predictions for $\bar{B} \to X_s\gamma$ in NLO QCD*, *Phys. Rev.* **D 58** (1998) 074004 [hep-ph/9802391];
P. Ciafaloni, A. Romanino and A. Strumia, *Two-loop QCD corrections to charged-higgs-mediated $b \to s\gamma$ decay*, *Nucl. Phys.* **B 524** (1998) 361 [hep-ph/9710312].

[10] S. Bertolini, F. Borzumati, A. Masiero and G. Ridolfi, *Effects of supergravity induced electroweak breaking on rare B decays and mixings*, *Nucl. Phys.* **B 353** (1991) 591;
R. Barbieri and G.F. Giudice, *$b \to s\gamma$ decay and supersymmetry*, *Phys. Lett.* **B 309**

(1993) 86 [hep-ph/9303270];
N. Oshimo, *Radiative b meson decay in supersymmetric models*, *Nucl. Phys.* **B 404** (1993) 20;
M.A. Diaz, *Constraints on supersymmetry due to $b \to s\gamma$: an improved calculation*, *Phys. Lett.* **B 304** (1993) 278 [hep-ph/9303280];
Y. Okada, *Light stop and the $b \to s\gamma$ process*, *Phys. Lett.* **B 315** (1993) 119 [hep-ph/9307249];
R. Garisto and J.N. Ng, *Supersymmetric $b \to s\gamma$ with large chargino contributions*, *Phys. Lett.* **B 315** (1993) 372 [hep-ph/9307301];
V. Barger, M.S. Berger, P. Ohmann and R.J. N. Phillips, *Constraints on SUSY GUT unification from $b \to s\gamma$ decay*, *Phys. Rev.* **D 51** (1995) 2438 [hep-ph/9407273].

[11] M. Ciuchini, G. Degrassi, P. Gambino and G.F. Giudice, *Next-to-leading QCD corrections to $B \to X_s\gamma$ in supersymmetry*, *Nucl. Phys.* **B 534** (1998) 3 [hep-ph/9806308].

[12] C. Bobeth, M. Misiak and J. Urban, *Matching conditions for $b \to s\gamma$ and $b \to s$ gluon in extensions of the standard model*, *Nucl. Phys.* **B 567** (2000) 153 [hep-ph/9904413].

[13] E. Gabrielli and G.F. Giudice, *Supersymmetric corrections to ε'/ε at the leading order in QCD and QED*, *Nucl. Phys.* **B 433** (1995) 3 [hep-lat/9407029].

[14] F. Borzumati, C. Greub, T. Hurth and D. Wyler, *Gluino contribution to radiative B decays: organization of QCD corrections and leading order results*, *Phys. Rev.* **D 62** (2000) 075005 [hep-ph/9911245].

[15] R. Rattazzi and U. Sarid, *Large tan(beta) in gauge-mediated SUSY-breaking models*, *Nucl. Phys.* **B 501** (1997) 297 [hep-ph/9612464];
H. Baer, M. Brhlik, D. Castano and X. Tata, *$b \to s\gamma$ constraints on the minimal supergravity model with large* $\tan\beta$, *Phys. Rev.* **D 58** (1998) 015007 [hep-ph/9712305];
T. Blazek and S. Raby, *$b \to s\gamma$ with large* $\tan\beta$ *in MSSM analysis constrained by a realistic* SO(10) *model*, *Phys. Rev.* **D 59** (1999) 095002 [hep-ph/9712257].

[16] L.J. Hall, R. Rattazzi and U. Sarid, *The top quark mass in supersymmetric* SO(10) *unification*, *Phys. Rev.* **D 50** (1994) 7048 [hep-ph/9306309].

[17] D.M. Pierce, J.A. Bagger, K. Matchev and R. jie Zhang, *Precision corrections in the minimal supersymmetric standard model*, *Nucl. Phys.* **B 491** (1997) 3 [hep-ph/9606211].

[18] M. Carena, D. Garcia, U. Nierste and C.E.M. Wagner, *Effective lagrangian for the $\bar{t}bH^+$ interaction in the MSSM and charged Higgs phenomenology*, *Nucl. Phys.* **B 577** (2000) 88 [hep-ph/9912516].

[19] H. Anlauf, *On the QCD corrections to $b \to s\gamma$ in supersymmetric models*, *Nucl. Phys.* **B 430** (1994) 245 [hep-ph/9406286].

[20] W. de Boer, M. Huber, A. Gladyshev and D.I. Kazakov, *The $b \to X_s\gamma$ decay rate in NLO, Higgs boson limits and LSP masses in the constrained minimal supersymmetric model*, `hep-ph/0007078`.